Homework 2 : Keplerian signatures in RV

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Objective : Know how to model a Keplerian Radial Velocity curve

1 Some important reminders

- Orbital elements of a Keplerian orbit

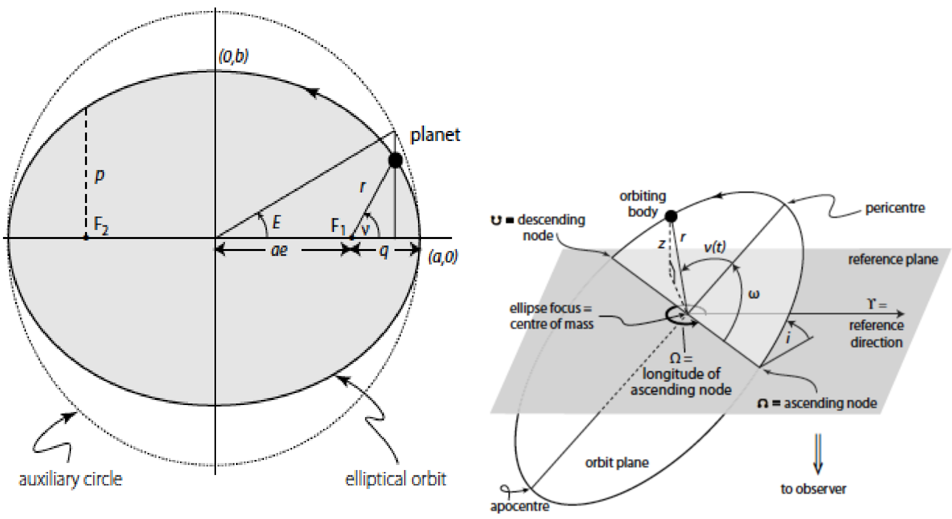

Figure 1: Figures extracted from [1]

- **Stellar** radial velocity:

$$\mathrm{RV(t)} = V_0 + K\Big[\cos\left(\omega+\nu(t)\right) + e\cos(\omega)\Big], \tag{1}$$

with K the semi-amplitude, e the eccentricity, ω the argument of periastron ($0 \leq \omega \leq 2\pi$), V_0 the velocity of the system's barycentre and $\nu(t)$ the true anomaly.

- RV semi-amplitude (in m.s^{-1}):

$$K = \left(\frac{2\pi G}{P}\right)^{\frac{1}{3}} \frac{M_p\ \sin(i)}{(M_\star + M_p)^{2/3}} \frac{1}{\sqrt{1-e^2}} \tag{2}$$

with P the planet period (in s), M_p the planet mass and $M_\star$ the stellar mass (in kg), i the inclination and e the eccentricity ($0 \leq e \leq 1$) of the planet's orbit.

- The true anomaly

$$\nu(t) = 2\ \arctan\left[\sqrt{\frac{1+e}{1-e}}\ \tan\left(\frac{E(t)}{2}\right)\right],$$

with $E(t)$ the eccentric anomaly which can be calculated using the mean anomaly $M(t)$:

$$M(t) = 2\pi\frac{t-t_0}{P} = E(t) - e\ \sin(E(t)), \tag{3}$$

with t_0 the time of the planet's passage at periastron. The first equality of (3) is calculated from the data, at a given t_0. The second equality is called the Kepler's equation and may be resolved by numerical iterative methods (e.g. Newton-Raphson algorithm).

- Multiple systems : RV signals can be approximated as a linear sum over N_p.

The Table.1 shows the RV semi-amplitude of the Solar System planets evaluated from (2).

Planets	Mean distance to the Sun (UA)	Periods (year)	Eccentricity	Mass ($M_\oplus$)	K ($m.s^{-1}$)
Mercury	0.38	0.24	0.206	0.06	0.008
Venus	0.72	0.61	$6.8\ 10^{-3}$	0.82	0.086
Earth	1.00	1.00	$1.67\ 10^{-2}$	1.00	0.089
Mars	1.52	1.88	$9.34\ 10^{-2}$	0.11	0.008
Jupiter	5.20	11.86	$4.85\ 10^{-2}$	317.8	12.47
Saturn	9.58	29.42	$5.56\ 10^{-2}$	95.2	2.758
Uranus	19.20	83.75	$4.60\ 10^{-2}$	14.6	0.299
Neptune	30.05	163.72	$1.1\ 10^{-2}$	17.2	0.281

Table 1: RV semi-amplitudes of the Solar System planets for $i = 90°$.

References

[1] Perryman, M., The exoplanet handbook, Chap.2, Cambridge University, 2011.

Exercice: Modelization of RV Keplerian signatures

The file 'table_SS.dat' contains the orbital parameters of the Solar System planets indicated in the column 2-5 of Table.1.

1. Model the RV Keplerian signatures of
   (a) Jupiter
   (b) The Earth
   (c) The whole solar system planets

   for $N = 2800$, $\Delta t = 22$ days, $V_0 = 0$ m/s, $t_0 = 0$ day, $\omega = \pi/4$ rad and $i = 90°$.

2. How each of the Keplerian parameters affect the RV curves ? Make a list.

3. Which kind of planetary targets are easier to detect ? Around which kind of stars ?

4. Effect of the orientation: evaluate the RV signatures of an Earth-like planet for
   $i = 0°, 45°, 90°$. Which is the optimal configuration ? Why ?

5. While the RV does not depend to the target distance, explain why the observation is more accurate when the observed star is close ?

Constantes: $G = 6.67 \times 10^{-11}$ m^3/kg/s^2, $M_\odot = 1.98 \times 10^{30}$ kg, $M_\oplus = 5.97 \times 10^{24}$ kg.

Help: Steps to solve (3) by a simple Newton-Raphson iteration

- Define $g(E) = E - e\sin(E) - M$ such as $g(E) = 0$ has a root at $E = E_0$.
- Assume an initial guess to E_0. A good value is $E_0 = M$ as E and M differ from a quantity close to e.
- Built the iteration scheme : $E_{i+1} = E_i - \frac{g(E_i)}{g'(E_i)}$ until $\left|\frac{g(E_i)}{g'(E_i)}\right| <$ to the required accuracy.