Calibrated Photometry

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Processing photometric data

When processing photometric data there are various steps that need to be followed. These are:

1. Data Reduction — bias, flat, dark
2. Correction for atmospheric extinction
3. Transformation to standard system
4. Heliocentric corrections — time-domain astronomy

The first step you have already seen during this `C2PU` METEOR, so we will not discuss that here. The final step is most important when considering time-domain astronomy, for example, determining orbital periods from light curves, or measuring oscillation periods. This will also not be discussed in this section. Here we will concentrate only on items 2 and 3. This involves working with reduced CCD images to obtain calibrated magnitudes which can then be compared with stellar or galactic models for astrophysical interpretation.

1 Atmospheric Extinction

Extinction is the absorption and scattering of electromagnetic radiation by dust and gas between a celestial object and the observer. Examples are interstellar extinction, extinction caused by a circumstellar disk, or extinction from the Earth's atmosphere. The effect of extinction is to attenuate the amount of flux that we receive from an object. If we wish to know information about the true brightness of the object, it is necessary to correct for extinction. In astronomy, the measured brightness of the star is that which is observed above the atmosphere. This is not the same as the *intrinsic* magnitude or brightness of the star, which are often referred to as the de-reddened magnitude.

In this session we will be discussing only atmospheric extinction, and we will be learning how to correct our observations for this. The main sources of atmospheric extinction is Rayleigh scattering by air molecules. This is proportional to λ^{-4} and it effects the blue part of the spectrum more than the red. Here λ = wavelength. Sources of atmospheric extinction include:

1. Rayleigh scattering ($\propto 1/\lambda^4$)
2. Molecular absorption
3. Absorption and scattering by aerosols ($\propto 1/\lambda^\alpha$, where $0 < \alpha < 1$)
4. Dust from deserts, volcanic ash, ...
5. Climate, altitude, ...

2 Understanding its effect

Atmospheric extinction is proportional to the length of the path which the light passes through. If the star is measured with a telescope on Earth, its light passes through many layers of the atmosphere. When the star is in the zenith the distance from the telescope to the top of the Earth's atmosphere is at its minimum. This is denoted as x in Fig. 1. The zenith angle is denoted as z. When $z = 0$ then the distance l that the light passes through is x. However it can be easily seen that if the star is at a different position in the sky the distance l that the light will pass through is greater and the light will be more attenuated. This distance can be written as $l = x/\cos(z) = x\sec(z)$.

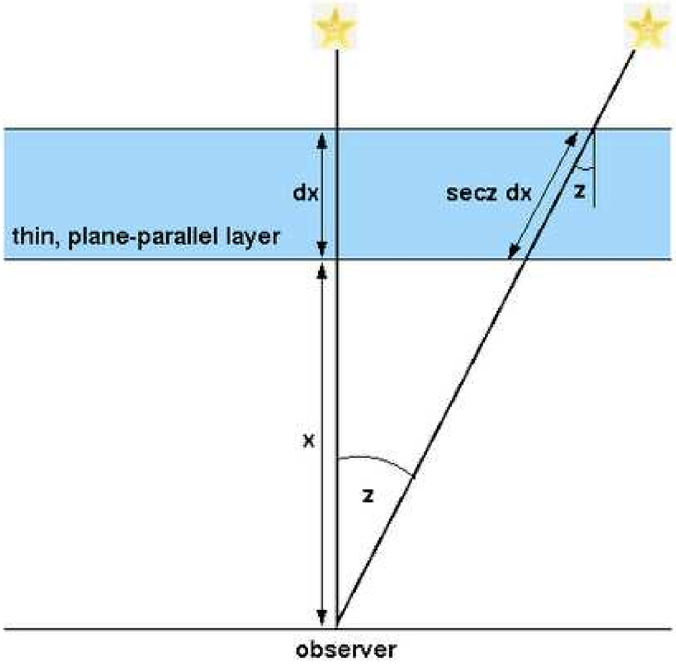

Figure 1: Starlight is attenuated as it passes through the atmosphere of thickness x. Figure taken from http://slittlefair.staff.shef.ac.uk/teaching/phy217/lectures/principles/L04/index.html

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The amount of light I_λ attenuated after passing through each dl ($= dx\sec(z)$ in Fig. 1) in the atmosphere is

$$\begin{aligned} dI_\lambda &= -\alpha_\lambda \; I_\lambda \; dl \\ &= -\alpha_\lambda(x) I_\lambda sec(z) dx \end{aligned} \tag{1}$$

where α_λ is extinction coefficient per unit length for a given wavelength. Rearranging we obtain

$$\frac{dI_\lambda}{I_\lambda} = -\alpha_\lambda(x) sec(z) dx \tag{2}$$

and this is a differential equation that we can solve for the full thickness of the atmosphere:

$$Ln(I_\lambda)|_{x=\infty}^{x=0} = - \; sec(z) \; \int_{x=\infty}^{x=0} \alpha_\lambda(x) dx = - \; sec(z) \; K'_\lambda \tag{3}$$

where we have defined $K'_\lambda \equiv \int_{x=\infty}^{x=0} \alpha_\lambda(x)dx$. This becomes

$$I_\lambda(x=0) = I_\lambda(x=\infty) \; e^{- \; sec(z) K'_\lambda} \tag{4}$$

and now by taking the logarithm and multiplying by –2.5 we obtain

$$-2.5 \log \left(\frac{I_\lambda(x=0)}{I_\lambda(x=\infty)} \right) = +2.5 \log(e) \;\; \sec(z) \; K'_\lambda) \tag{5}$$

The left hand side of this equation is the difference in magnitudes of the object observed at the telescope, m_λ^{obs}, and outside the atmosphere, $m_\lambda(0)$. Here 'obs' refers to the 'observation'.

$$m_\lambda^{obs} - m_\lambda(0) = 1.086 \sec(z) \; K'_\lambda \equiv \sec(z) K_\lambda \equiv X K_\lambda \tag{6}$$

K_λ is the integrated atmospheric extinction coefficient, **given in magnitudes per unit of air mass**. We arrive at the following equation:

$$m_\lambda^{obs} = m_\lambda(0) + K_\lambda X \tag{7}$$

Note that I always use the subscript λ to emphasize that this value is very dependent on the wavelength in which we observe. So this expression is only valid for monochromatic observations or narrow-band observations. For this TP we will make this assumption. If we use broad-band photometry, however, then the expression to consider is no longer dependent only on λ. It becomes

$$m_{\lambda_{\text{eff}}}^{obj} = m_{\lambda\text{eff}}(0) + K_{0,\lambda_{\text{eff}}} X + K_{1,\lambda_{\text{eff}}}(\lambda_1 - \lambda_2) X \tag{8}$$

where λ_{eff} is the effective or central wavelength of the filter, $(\lambda_1 - \lambda_2)$ refers to a colour of the star observed in filters 1 and 2, and now we have two coefficients of interstellar extinction, K_0 and K_1 which depend on the effective wavelength.

3 Air Mass

Airmass is the geometric path of light from an astrophysical object to the observer, that is, by passing through the Earth's atmosphere. At $z = 0$ the airmass is equal to 1 unit. At angles different to $z = 0$ the airmass is proportional to $l \equiv x \sec(z)$. In order to correct the flux from an object for atmospheric extinction, we need to know its zenith angle. The angle depends on the coordinates of the object, the location of the observatory and the time at which we observe. It can be calculated directly from this equation:

$$\sec(z) = [\sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(H)]^{-1} \tag{9}$$

Here ϕ = latitude of the observatory, H = hour angle of the object, where $H = \text{TS} - \alpha$, TS is sidereal time and α and δ are the equatorial coordinates of the object.

If $z \leq 60°$ then we can approximate the airmass by

$$X \simeq \sec(z). \tag{10}$$

However, at greater angles, $z > 60°$, then this approximation is no longer valid and we should use the following equation to calculate the airmass X:

$$X \simeq \sec(z) - 0.0018167(\sec(z) - 1) - 0.002875(\sec(z) - 1)^2 - 0.0008083(\sec(z) - 1)^3 \tag{11}$$

4 Calculation of coefficients

As we discussed earlier, atmospheric extinction depends on a number of factors. Many of these factors are, however, unchangeable for a given observing location (climate, altitude etc. does not change). If we wish to calculate the extinction coefficients for a night of observations, then we are implicitely making an assumption that they are non-changing during the observing period. This means that night observing conditions need to be very stable and *photometric*, as the weather and sky conditions are the only variables for a given location.

However, we also just saw that the location of the object in the sky, in particular its zenith angle and the path length the light will traverse in the atmosphere, will also effect the attenuation of the flux. But we know that this behaves as $K_\lambda X$ where X is the air mass and K_λ is the coefficient of atmospheric extinction. If we take measurements over the course of an hour and the object is rising in the sky, the observed flux will increase for the same exposure time.

In the ideal case we can assume that we have no errors on the measurements. Therefore with only two measurements of a source, corrected for exposure time, we can calculate the coefficient(s) directly by using Eq. 7.

However, our measurements have errors and any fluctuations in the observing conditions will cause a fluctuation in the incoming flux. Therefore we need to observe our star (or many stars) many times over the duration of the observations in order to obtain a reliable estimate of the incoming flux at different air masses.

If we plot the observed magnitudes of the star(s) as a function of air mass (X_i) we obtain a series of points that can be fitted by a straight line. The slope of the line is K_λ in magnitudes per airmass. and extrapolating the line until $X = 0$ we obtain the *instrumental magnitude*, $m_\lambda(0)$. This is the magnitude that the object would have if it were observed at the top of the Earth's atmosphere (without considering interstellar extinction).

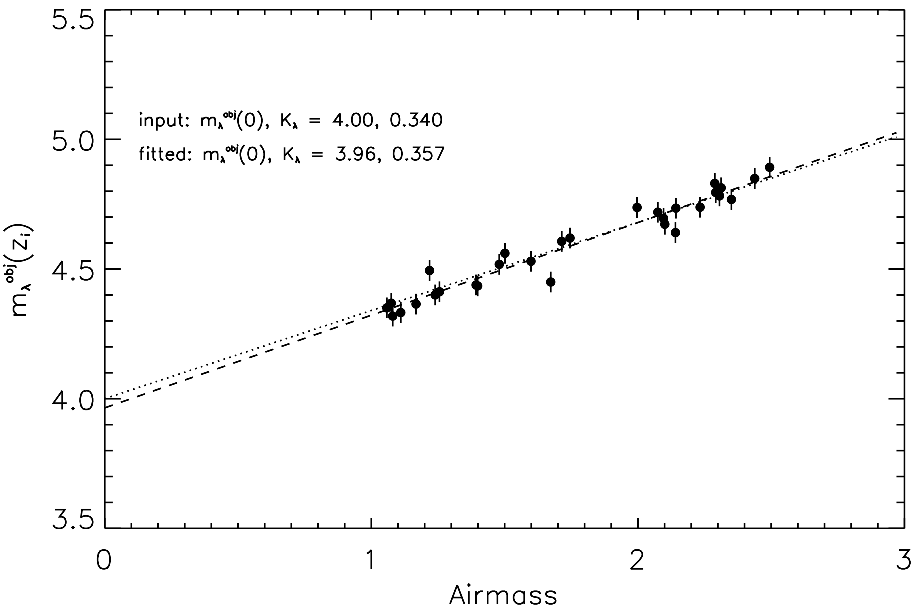

Figure 2: Measured magnitudes of a given object observed at different airmasses. A linear fit to the data will give the slope and the intercept of the line. The slope is the coefficient of atmospheric extinction, K_λ, and the intercept is the magnitude corrected for extinction, $m_\lambda(0)$. Here we have supposed a total of 30 measurements with a mean magnitude error of 0.04 mag. Even with a high number of measurements and reasonable errors on the magnitudes we still obtain an offset in the extinction coefficient and the instrumental magnitude.

5 Conversion to absolute scale

The instrumental magnitude depends on the characteristics of the instrument that was used to observe the object. It is only useful when we are comparing objects observed with the same observing setup. For certain astrophysical problems it is necessary to convert these values to an absolute scale. To do this we need to know the *zero-points*, or the reference values that allows one to convert from our instrumental magnitude to a standard scale. This is indeed necessary when working with clusters. It is only by comparison with models and other observations that we can then deduce astrophysical information from the observations, such as distance to the cluster or turn-off mass.

The definition of magnitude is $m_1 - m_2 = -2.5 \log \frac{f_1}{f_2}$, and up to know we have only measured m_1, where the 1 refers to the object that we are interested in measuring. From here on we will denote this as *obj*. To calculate the zero-point we need to observe a *standard star* whose photmetric magnitude is known. We will refer to this star (or stars) as the standard star (m_2) and use the superscript *std*. By repeating Sect. 4 above we can deduce the instrumental magnitude of the standard star, $m_\lambda^{std}(0)$,

$$m_\lambda^{std}(0) = m_\lambda^{std}(X_i) - K_\lambda X_i, \tag{12}$$

where X_i refer to the observations in the i^{th} airmass. Now we have

$$m_\lambda^{obj}(0) - m_\lambda^{std}(0) \equiv -2.5 \log \left(\frac{S_\lambda^{obj}(0)}{S_\lambda^{std}(0)} \right) \equiv -2.5 \log \left(\frac{f_\lambda^{obj}}{f_\lambda^{std}} \right) \equiv m_\lambda^{cal,obj}(0) - m_\lambda^{cal,std}(0) \tag{13}$$

where $m_\lambda^{cal,obj}$ and $m_\lambda^{cal,std}$ are the calibrated photometric magnitude of the object and the standard, respectively. To determine $m_\lambda^{cal,obj}$ we have all that we need:

$$m_\lambda^{cal,obj} = m_\lambda^{cal,std} + [m_\lambda^{obj}(0) - m_\lambda^{std}(0)] \tag{14}$$

The latter two terms have been measured and $m_\lambda^{cal,std}$ is obtained from standard star catalogues. This same equation is applied to all of the stars that we measure to obtain the calibrated photometric magnitudes of the stars in the cluster.

For a better precision on the data one can use several standard stars, and in the ideal case, the standard star is located in the object observing field. As the latter condition is not always possible a standard star should be chosen to be as close as possible in distance from the observing field. If the filter is broad then the standard and the objects should also be of the same spectral type.

6 Generalisation of data calibration for polychromatic observations

We have seen that the coefficients of atmospheric extinction depend on wavelength. The extinction can be corrected for by using a linear approximation if the observations are monochromatic or the filters are narrow enough to neglect second and higher order terms. This is what we will assume for our observations. However, as we are using broad-band filters (similar to sloan g, r, i) we should take into account the higher order terms. The calibration can be done as two separate steps: 1. correction for atmospheric extinction, and 2. transformation to a standard system, or the calibration can be done by solving these two problems simultaneously. In this section we give the general equations to solve the system in two steps.

We require having a set of M standard stars observed in $X_{i=1,..,N}$ airmasses. We use the following general equation to derive the instrumental magnitude of the j^{th} standard star corrected for atmospheric extinction

$$m_\lambda^{std,j}(X_i) = m_\lambda^{std,j}(0) + K_{0,\lambda}X_i + K_{1,\lambda}X_i(\lambda_m - \lambda_k)_{std,j} + ... \qquad (15)$$

Here $K_{0,\lambda}$ and $K_{1,\lambda}$ are the two (or more) extinction coefficients to determine and $(\lambda_m - \lambda_k)_{std,j}$ is the colour of the j^{th} standard star, assumed to be known. An example is the colour $(U - B)$. As we have N observations of M stars in different airmasses, $m_\lambda^{std,j}(X_i)$, we can resolve the set of $M \times N$ equations using an optimization method to determine the two coefficients and the corrected instrumental magnitudes of the standards, $m_\lambda^{std,j}(0)$. Once the coefficients are determined we apply them to the observations of the science targets ^{obs,j} to determine the corrected instrumental magnitudes, $m_\lambda^{obj,j}(0)$.

$$m_\lambda^{obj,j}(0) = m_\lambda^{obj,j}(X_i) - K_{0,\lambda}X_i - K_{1,\lambda}X_i(\lambda_m - \lambda_k)^{obj,j} + ... \qquad (16)$$

To transform the instrumental magnitudes to standards we need to determine a set of polynomial coefficients to allow us to do this. We use the standard star, and knowing its calibrated magnitude we write a set of equations so that we can solve for the coefficients.

$$m_{\lambda_k}^{std,j}(0) = m_{\lambda_k}^{cal,std,j} + a_{0,\lambda_k} + a_{1,\lambda_k}(m_{\lambda_l} - m_{\lambda_m})_{cal,std,j} + a_{2,\lambda_k}(m_{\lambda_l} - m_{\lambda_m})^2_{cal,std,j} + ... \qquad (17)$$

where the LHS contains the corrected instrumental magnitudes that we have already determined, $m_{\lambda_k}^{cal,std,j}$ and $(m_{\lambda_l} - m_{\lambda_m})_{cal,std,j}$ are the known (catalogued) photometric magnitude and colour of the j^{th} standard. We do this for the M standard stars and solve the system to determine the coefficients a_{0,λ_k}, a_{1,λ_k}, a_{2,λ_k}, etc. Once we have determined these coefficients we can simply apply a similar equation to the science targets to derive the calibrated instrumental magnitude $m_{\lambda_k}^{cal,obj,j}$ of the j^{th} target star:

$$m_{\lambda_k}^{cal,obj,j}(0) = m_{\lambda_k}^{obj,j} - a_{0,\lambda_k} - a_{1,\lambda_k}(m_{\lambda_l} - m_{\lambda_m})_{cal,obj,j} - a_{2,\lambda_k}(m_{\lambda_l} - m_{\lambda_m})^2_{cal,obj,j} + ... \qquad (18)$$

This is repeated for all of the filters used.

A few notes:
— The number of coefficients that we can determine depends on the number of measured standard stars. The more standards used, the more coefficients can be derived, and the higher precision in the transformation.
— For assymetric filters, colour-dependent terms are necessary.
— One should choose calibration stars covering a wide range of colours.
— It is also possible to solve for the atmospheric extinction and the transformation to the standard system for all of the filter bands used at the same time.