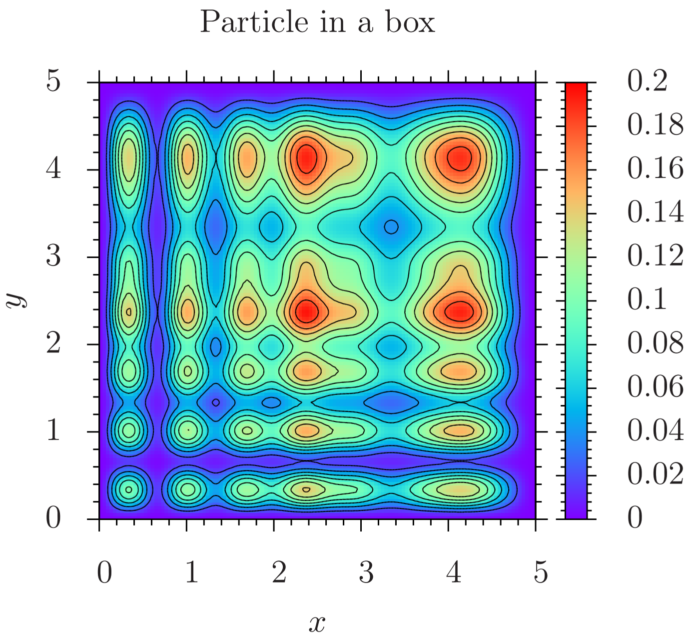
FC1.5 Numerical methods



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Bibliography / links

- A First Course in Numerical Analysis, Ralston, A. and Rabinowitz, P., Dover Publications, Inc., 2001
- Finite difference methods for differential equations, LeVeque, R.J., 2007
- Finite difference and spectral methods for ordinary and partial differential equations, Trefethen, L.N., 1996
- Numerical methods for engineers and scientists, Hoffman, Joe D and Frankel, Steven, CRC press, 2001
- Finite volume methods for hyperbolic problems, LeVeque, Randall J, Cambridge university press, 2002

Contents

When elaborating a mathematical model of a physical problem chances are that one ends up with differential equations, either Ordinary Differential Equation (ODE) or Partial Differential Equation (PDE). In astrophysics the need for numerical simulations is even more crucial since the objects under study are far from being located in the lab. A large variety of physical problems are described in that way such as e.g. fluid dynamics, heat conduction, n-body simulation, radiative transfer, etc. Solving the equations of mathematical physics numerically is a good playground to face the classical problems of numerical analysis such as interpolating, integration, solving linear system of equations, studying the stability of a numerical schemme etc . . . . The student will acquire all the necessary theoretical background to solve differential equations numerically and will apply the freshly acquired material in small illustrative computer projects.

Part 1: Fundamentals

by G. NICCOLINI

1. Round-off error and the floating point number system.
2. Truncation errors.
3. Interpolation
   (a) Lagrange interpolation and related error
4. Integration of functions
   (a) Order of accuracy
   (b) Newton-Cotes formulae
5. Linear systems of equations
   (a) Iterative methods for sparse systems (Jacobi, Gauss-Seidel, Successive Over-Relaxation, conjugate gradient, . . .)
6. Finite Difference (FD) approximations of differential operators
   (a) How to derive FD formulae
   (b) Truncation errors

Part 2: Partial/Ordinary Differential Equations

by G. NICCOLINI

1. Solving ODE numerically
   (a) Solving ODE numerically
      - Mathematical properties
      - One-step methods (Euler, RK, Taylor series, explicit vs implicit methods)
      - Multi-step methods (Leap-Frog, Adams-Moulton, Adams-Bashforth)
      - Stability analysis : notion of zero and absolute stability, characteristic polynomial
   (a) Solving PDEs
      - Mathematical properties
      - PDEs as vector ODEs
      - Boundary conditions
      - Truncation error and consistency
      - Convergence and stability

- Recalling the Discrete Fourier Transform
- Von Neumann stability criterion

Part 3: Practical works

by R. Hassani, D. Laveder

1. Short intro to programming.Development of computer programs to solve linear PDE of mathematical physics (wave equation, SchrÃűdinger, diffusion, advection)
2. A first (tiny) step toward CFD : solving the Burgers equation with the Mac Cormack scheme